STAT 509 2017 Summer Exam 1 Formula Sheet

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- **Binomial:** $E(Y) = np$ and $\text{var}(Y) = np(1-p)$.

$$p_Y(y) = \begin{cases} \binom{n}{y} p^y (1-p)^{n-y} & \text{for } y = 0, 1, 2, \ldots, n, \\ 0 & \text{otherwise.} \end{cases}$$

- **Geometric:** $E(Y) = \frac{1}{p}$ and $\text{var}(Y) = \frac{1-p}{p^2}$.

$$p_Y(y) = \begin{cases} (1-p)^{y-1} p & y = 1, 2, 3, \ldots \\ 0 & \text{otherwise.} \end{cases}$$

- **Negative Binomial:** $E(Y) = \frac{r}{p}$ and $\text{var}(Y) = \frac{r(1-p)}{p^2}$.

$$p_Y(y) = \begin{cases} \binom{y-1}{r-1} p^r (1-p)^{y-r} & y = 1, 2, 3, \ldots \\ 0 & \text{otherwise.} \end{cases}$$

- **Hypergeometric:** $E(Y) = n(\frac{r}{N})$ and $\text{var}(Y) = n(\frac{r}{N})(\frac{N-r}{N})(\frac{N-n}{N-1})$.

$$p_Y(y) = \begin{cases} \frac{\binom{r}{y}\binom{N-r}{n-y}}{\binom{N}{n}}, & y \le r \text{ and } n - y \le N - r \\ 0, & \text{otherwise.} \end{cases}$$

- **Poisson:** $E(Y) = \lambda t$ and $\text{var}(Y) = \lambda t$.

$$p_Y(y) = \begin{cases} \frac{(\lambda t)^y e^{-\lambda t}}{y!}, & y = 0, 1, 2, \ldots \\ 0, & \text{otherwise.} \end{cases}$$

- **Exponential:** $E(Y) = \frac{1}{\lambda}$ and $\text{var}(Y) = \frac{1}{\lambda^2}$.

$$f_Y(y) = \lambda e^{-\lambda y}, \quad y \ge 0$$
$$F_Y(y) = 1 - e^{-\lambda y}, \quad y \ge 0.$$

- **Weibulll:** $E(Y) = \delta \Gamma\left(1 + \frac{1}{\beta}\right)$ and $\text{var}(Y) = \delta^2 \left[\Gamma\left(1 + \frac{2}{\beta}\right) - \left(\Gamma\left(1 + \frac{1}{\beta}\right)\right)^2\right]$.

$$f_Y(y) = \frac{\beta}{\delta}\left(\frac{y}{\delta}\right)^{\beta-1} e^{-(y/\delta)^\beta}, \quad y \ge 0$$
$$F_Y(y) = 1 - e^{-(y/\delta)^\beta}, \quad y \ge 0.$$

- **Normal:** $E(Y) = \mu$ and $\mathrm{var}(Y) = \sigma^2$.

$$f_Y(y) = \frac{1}{\sqrt{2\pi}\sigma} e^{-\frac{(y-\mu)^2}{2\sigma^2}}, \quad -\infty < y < \infty$$

- Construct confidence interval: **point estimate** $\pm$ **margin of error**:
  - Population proportion p: $\hat{p} \pm z_{\frac{\alpha}{2}} \sqrt{\frac{\hat{p}(1-\hat{p})}{n}}$
- 4-step procedure to construct hypothesis testing:
  1. State H_0 and H_a
  2. Calculate test statistic:
     * Population proportion: $z_0 = \frac{\hat{p} - p_0}{\sqrt{\frac{p_0(1-p_0)}{n}}}$
  3. Calculate p-value

$$\begin{aligned} H_a : p > p_0 : &\quad P(Z > z_0) \\ H_a : p < p_0 : &\quad P(Z < z_0) \\ H_a : p \neq p_0 : &\quad 2P(Z < -|z_0|) \end{aligned}$$

  4. Make decision and state conclusion:
     * p-value $\leq \alpha \Longrightarrow$ Reject H_0
     * p-value $> \alpha \Longrightarrow$ Fail to reject H_0